Natural Convection With Mercury in a Uniformly Heated Vertical Channel During Unstable Laminar and Transitional Flow

Experimental heat transfer correlations were determined for natural convection in mercury in a uniformly heated vertical channel with width-to-height ratios varying from 0.25 to infinity (the single plate limit). Local modified Grashof numbers varied between 10^8 and 10^{11}. Temperature and velocity profiles were obtained for these same channel configurations and local Grashof numbers. Instantaneous velocity plots indicate the range of conditions studied to proceed from unstable laminar flow well into the transition regime. Heat transfer results are in good agreement with earlier results reported for stable laminar flow which were limited to values of modified Grashof number less than 10^9.

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SCOPE

Liquid metals are of great present-day interest due to their unique heat transfer capabilities, particularly so as coolants in the Liquid Metal Fast Breeder Reactor (LMFBR) program which is a high priority activity of the U.S. Atomic Energy Commission. Upon loss of forced flow, as in the case of pump failure, the heat transfer mode from the reactor core area would be one of natural convection with the liquid metal coolant serving as the heat transfer medium. An analysis of safety hazards associated with such an occurrence has been difficult owing to a general lack of fundamental experimental data necessary to confirm analytical models presently used.

Listed in Table 1 are some comparative properties for air, water, mercury, and sodium. It is apparent from the values listed that the liquid metals, mercury and sodium, have the superior thermal conductivities compared with air and water and that they display superior high-temperature capabilities.

Liquid sodium is the most desirable of the substances listed for high-temperature service. Due to the extra safeguards required to perform experiments with sodium, because of its corrosive and reactive nature, most experiments with liquid metals employ mercury. Mercury must also be handled with care but this is much less a problem than for sodium.

The trends in heat transfer observed in mercury should be representative of sodium and other low Prandtl number fluids. While no comparisons of liquid metal heat transfer results have been made for natural convection between different metals, Johnson et al. (1954) have indicated that due to different wetting characteristics of mercury, heat transfer results for mercury average about 38% lower than for other liquid metals in forced convection. The use of heat transfer results from mercury for sodium would probably therefore give conservative design information.

The first objective of this work was to extend the range of heat transfer results two orders-of-magnitude in the local Grashof number beyond those available in the literature. Previous experiments have terminated at values of $Gr_x^* = 10^9$. At this value of Gr_x^*, indications were that the flow regime in the boundary layer was stable laminar flow with some evidence of unstable laminar flow near $Gr_x^* = 10^9$. Of interest was the nature of the heat exchange mechanism in the unstable-laminar-flow as well as the laminar-to-turbulent transition regime. Correlations of the dimensionless heat transfer coefficient, the Nusselt number Nu_x, vs. Gr_x^* were desired in the range $10^9 < Gr_x^* < 10^{11}$ for a single heated vertical plate and for a symmetrically-heated vertical channel with varying width-to-height ratios.

The second objective was to obtain temperature and velocity profiles at each channel width including the single plate case. From such results the location and magnitude of the peak velocities might be determined as well as an indication of the growth and extent of the thermal and hydrodynamic boundary layers.

The final objective was to measure instantaneous temperatures and velocities over the range of experimental conditions so that some quantification of instability and transition might be accomplished. Measurements of this sort in liquid metals were first obtained by Colwell (1974). The present effort extends the Grashof number range two orders of magnitude beyond that studied by Colwell.

Analytical studies of natural convection with low Prandtl number fluids include those of Sparrow and Gregg (1959) who reported results of a similarity analysis for low Prandtl number fluids adjacent to an isothermal heated vertical plate. Chang et al. (1964) used a perturbation analysis to obtain heat transfer results for liquid metals in natural convection adjacent to a single uniformly heated vertical plane surface. Cygan and Richardson (1968) and Kuiken (1969) also achieved analytical

TABLE 1. COMPARATIVE PROPERTIES FOR AIR, WATER, MERCURY, SODIUM

Material	Thermal conductivity, k, W/m K @ 366.7 K	Melting point, K	Boiling point, K	Prandtl number Pr (dimensionless) @ 366.7 K
Air	0.0310			0.694
Water	0.678	273	373	1.91
Mercury	9.08	234	630	0.0191
Sodium	86.2	371	1162	0.0118

results for the heated isothermal plate with a low Prandtl number fluid in natural flow.

Julian and Akins (1969) presented experimental results for an electrically heated vertical plate in mercury. Colwell and Welty (1973) were the first to perform experiments in a vertical channel between heated plane surfaces. In addition to developing correlations for the Nusselt number as a function both of the modified Grashof number and channel width, they also obtained velocity data using a quartz-coated hot film anemometer. Colwell and Welty found that, in the case of mercury, the Nusselt number increased as channel width decreased with the maximum heat transfer capability existing for a channel width-to-height ratio near 0.10. This result had not been reported previously and is, apparently, a unique characteristic of low Prandtl number fluids.

CONCLUSIONS AND SIGNIFICANCE

Heat transfer data were obtained for a single, uniformly heated vertical plate in the range $10^6 < Gr_x^* < 10^{11}$. This range includes stable laminar flow, unstable laminar flow, and laminar-to-turbulent transition. For a Prandtl number of 0.023, that of mercury at the experimental conditions used, the resulting nondimensional correlation was

$$Nu_x = 0.196\ Gr_x^{*0.188} \tag{1}$$

This expression is identical to that reported previously by Julian and Akins (1969) for stable laminar flow. A modified form of the above expression with the exponent on the Grashof number forced to be a more convenient value is

$$Nu_x = 0.150\ Gr_x^{*0.2} \tag{2}$$

Comparisons of these relations with those reported in the literature at lower modified Grashof numbers reveal no significant variance. A single correlation such as Equation (2) may be used over the entire Grashof number range.

Measurements in a uniformly and symmetrically heated vertical channel at width-to-height ratios of 0.25, 0.50, and 0.67 spanned the range $10^8 < Gr_x^* < 10^{11}$ which is two orders of magnitude higher in the modified Grashof number than previously reported. Heat transfer correlations of the data are

$$Nu_x = 0.256\ Gr_x^{*0.178}, \quad W/L = 0.25 \tag{3}$$

$$Nu_x = 0.252\ Gr_x^{*0.178}, \quad W/L = 0.50 \tag{4}$$

$$Nu_x = 0.249\ Gr_x^{*0.178}, \quad W/L = 0.67 \tag{5}$$

and, with all data correlated without regard for the channel spacing, W/L, the single expression is

$$Nu_x = 0.252\ Gr_x^{*0.178} \tag{6}$$

The effect of channel width, although slight, is nonetheless consistent with the results of Colwell and Welty (1973) who reported an increase in heat transfer capability as channel width was decreased, at least over the range of channel spacings studied. Experimental limitations precluded narrower spacing in this work.

Local temperature and velocity data revealed profiles with characteristic natural convective tendencies. As the channel became narrower, the maximum velocity decreased. Consistent with the previously-mentioned increase in heat transfer coefficient with decreased channel width the wall temperature was lower for narrower channels. Obvious temperature stratification in mid channel was noted for smaller channel widths.

Flow instability measurements in the form of instantaneous velocity data using the hot-film anemometer revealed quantitative information regarding the nature of the flow field. These measurements represent the first reported data of this sort for liquid metals.

PREVIOUS WORK

Previous studies for natural convection with liquid metals adjacent to a uniformly heated vertical plate have been both analytical and experimental in nature. Experimental studies have covered both single vertical-plate and channel configurations whereas analytical studies have been limited to the single vertical plate. Analytical solutions are further limited by the assumption of laminar flow, whereas experimental results are valid over the range of data achieved.

Historically, analytical studies preceded experiments. Sparrow and Gregg (1956) presented an exact similarity solution to the coupled momentum and energy equations for the uniform flux boundary condition. They presented solutions for Prandtl numbers of 0.1, 1, 10, and 100, but because of computer limitations did not extend their results outside this range. Their heat transfer correlation $Nu_x/Gr_x^{*0.2} = f(Pr)$ can be extrapolated to a Prandtl number of 0.023, the Prandtl number of mercury at room temperature as used in the present work. The resulting correlation is

$$Nu_x = 0.16\ Gr_x^{*0.2} \tag{7}$$

In a later paper Sparrow and Gregg (1959) extended their similarity solution to lower Prandtl numbers for the isothermal surface boundary condition but not for the uniform flux condition. Since the temperature of a uniform flux plate varies with the vertical direction, x, as $x^{1/5}$, the isothermal surface condition is sufficiently similar that solutions are worth comparing. The results of Sparrow and Gregg for the isothermal plate, using a linear interpolation for application to a Prandtl number of 0.023, and expressed in terms of the modified Grashof number are

$$Nu_x = 0.138\ Gr_x^{*0.2} \tag{8}$$

Chang et al. (1965) used a perturbation scheme to solve the coupled momentum and energy equations for the uniform flux boundary condition for low Prandtl numbers. Their heat transfer results were

$$Nu_x = 0.632\ Pr^{0.37}\ Gr_x^{*0.2} \tag{9}$$

which, for a Prandtl number of 0.023 becomes

$$Nu_x = 0.154\ Gr_x^{*0.2} \tag{10}$$

Cygan and Richardson (1968) reported heat transfer and skin friction results for a vertical isothermal plate using approximate representations of the velocity and temperaure profiles. They represented the velocity profile as the difference of two exponential terms representing, in turn, viscous and inviscid regions of the boundary layer, and represented the temperature profile with a complimentary error function term. Their heat transfer results, linearly interpolated to a Prandtl number of 0.023 and expressed in terms of the modified Grashof number, are

$$Nu_x = 0.134\ Gr_x^{*0.2} \tag{11}$$

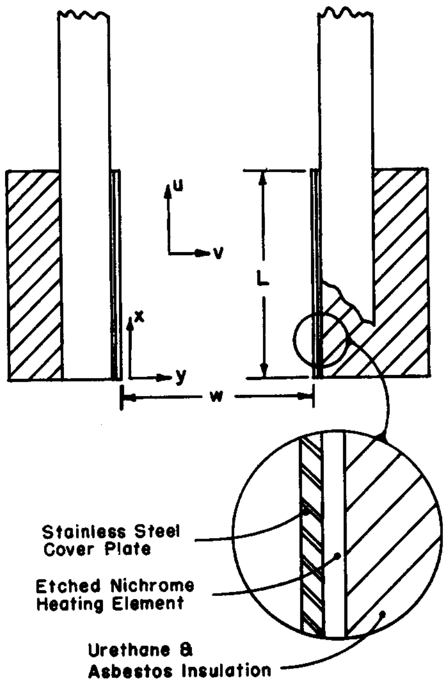

Fig. 1. Schematic diagram of test configuration.

Kuiken (1969) used the method of matched asymptotic expansions to integrate the coupled momentum and energy equations for the limiting case as the Prandtl number approached zero. His heat transfer results for the isothermal surface boundary conditions, once again linearly interpolated for a Prandtl number of 0.023 and expressed in terms of the modified Grashof number are

$$Nu_x = 0.137\ Gr_x^{*0.2} \tag{12}$$

Julian and Akins (1969) reported an experimental study of natural convection from a constant flux vertical surface in mercury. They presented heat transfer results for a modified Grashof number range between 10^4 and 10^9, from data taken at vertical positions ranging from 0.635 to 4.445 cm from the leading edge. Their heat transfer results were

$$Nu_x = 0.196\ Gr_x^{*0.188}, \quad 10^4 < Gr_x^* < 10^9 \tag{13}$$

Most recently Colwell and Welty (1973) performed an experimental study of natural convection from both a single vertical plate and a channel composed of two vertical plates; in each case the plates were uniform flux surfaces. They presented temperature and velocity profiles, and heat transfer data for a modified Grashof number range of 10^4 to 10^9. Their heat transfer results for the single vertical plate were

$$Nu_x = 0.23\ Gr_x^{*0.18}, \quad 10^4 < Gr_x^* < 10^9 \tag{14}$$

Colwell and Welty also presented similar heat transfer results for the vertical channel, where their channel width-to-height ratio W/L varied from 0.50 to 0.056. Their results, including the channel spacing in the correlation, were

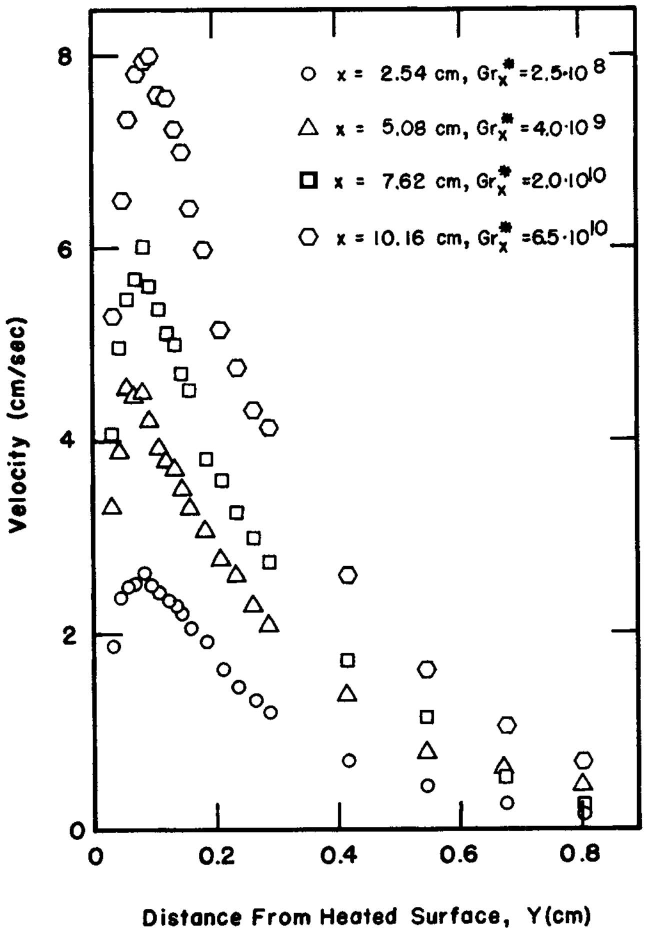

Fig. 2. Velocity profiles for single heated plate at four vertical positions.

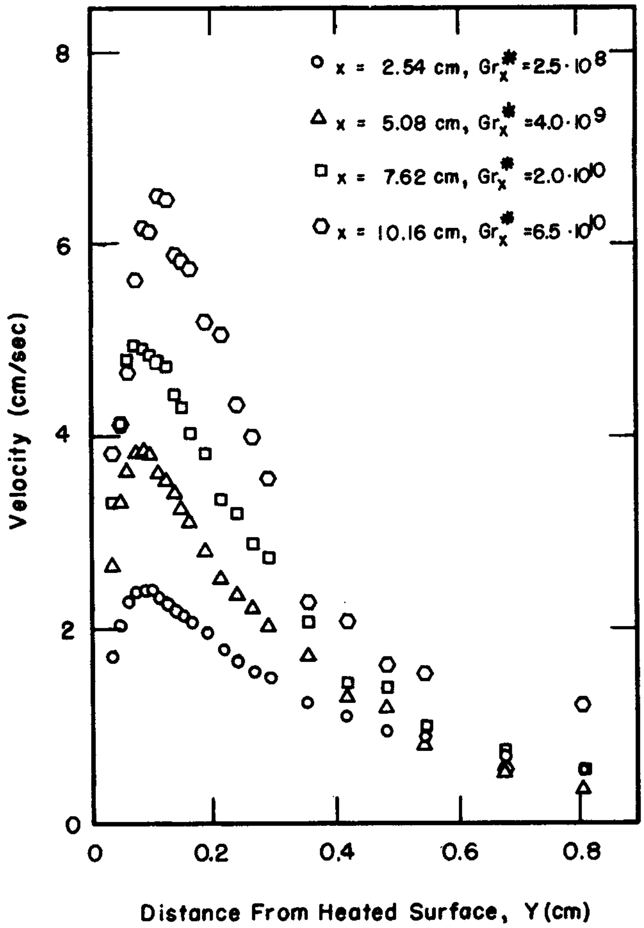

Fig. 3. Velocity profiles for vertical channel, $W/L = 0.25$, at four vertical positions.

$$Nu_x(W/L) = 0.268\ [Gr_x^*(W/L)^5]^{0.165},$$
$$10^3 < Gr_x^*(W/L)^5 < 10^9 \quad (15)$$

Of more interest even than the actual magnitude of their results was the trend they discovered as the channel spacing was varied. They found that the Nusselt number at a constant Grashof number increased as the channel spacing decreased until the channel width-to-height ratio W/L reached a value between 0.10 and 0.05 where the viscous effects of the plates finally dominated and the Nusselt number dropped off very rapidly as the channel spacing continued to decrease. This indicated the existence of an optimum channel spacing to maximize the heat transfer or minimize surface temperature with liquid metals which is not the case for other liquids.

EXPERIMENTAL METHODS

The data for this paper were taken in a symmetrically heated vertical channel. The channel configuration is shown schematically with the conventional coordinate system in Figure 1. The channel had a vertical length L of 14.70 cm and the width W was varied from 3.675 to 9.80 cm. The channel was open at the top and bottom but was enclosed on each side by 0.368-cm thick plastic side plates. The back side of each uniformly heated plate was insulated with 2.94 cm of asbestos and 2.94 cm of urethane foam so that the heat loss from the back sides of the plates was less than 0.2%. As a separate situation, one of the heated plates and both side plates were removed and data were taken for a single vertical plate.

Temperatures were measured with an iron-constantan subminiature grounded thermocouple, manufactured by Omega Engineering, Inc. The velocities were measured with a constant temperature hot-film anemometer system, manufactured by Thermo-Systems, Inc.*

RESULTS AND DISCUSSION

In the discussion of heat transfer it is of interest to know orders of magnitude of both temperature differences and velocity. For modeling this flow situation with any kind of empirical expression, the actual shapes of the temperature and velocity profiles are of interest. Also, since the heat transfer data for this paper were taken during unstable laminar flow and transition, it is important to have a general idea of the characteristics of the instability. These matters will now be discussed.

Velocity and temperature profiles were taken for the single vertical plate and for three vertical channel spacings, W/L = 0.25, 0.50, and 0.67. The ambient temperature was maintained at approximately 27.8°C, and heat flux was held constant at 3.31×10^4 J/m² s, while profiles were taken at vertical locations of 2.54, 5.08, 7.62, and 10.16 cm.

Dimensional velocities are presented here for the single plate in Figure 2 and for the narrowest channel spacing, W/L = 0.25, in Figure 3. There were no large changes in the profiles as the position of the second plate was changed, but the maximum velocity at the higher vertical positions seemed to decrease from the single plate case to the smallest channel spacing by 15 to 20%.

In a similar manner, the dimensional temperature pro-

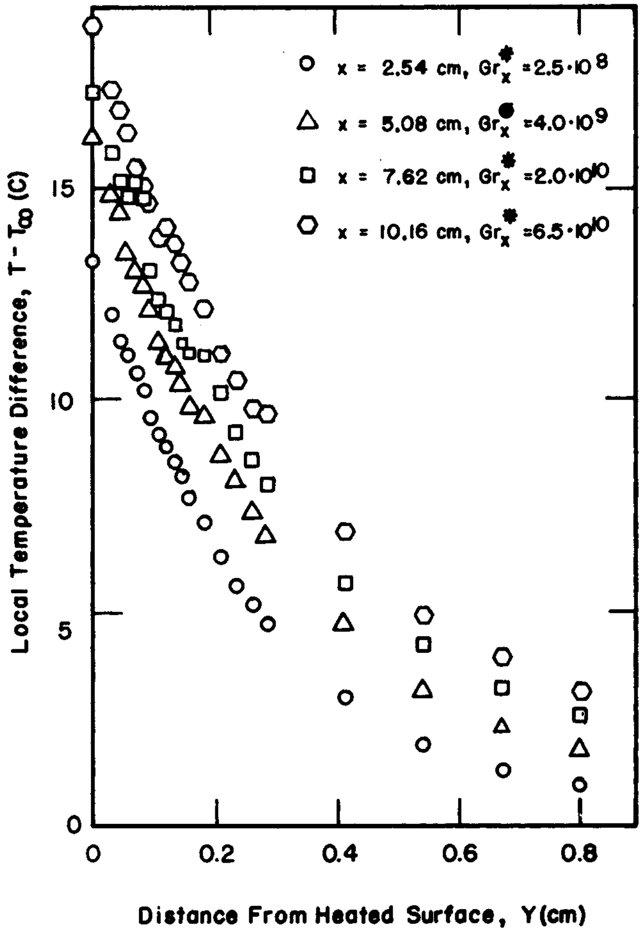

Fig. 4. Temperature profiles for single heated plate at four vertical positions.

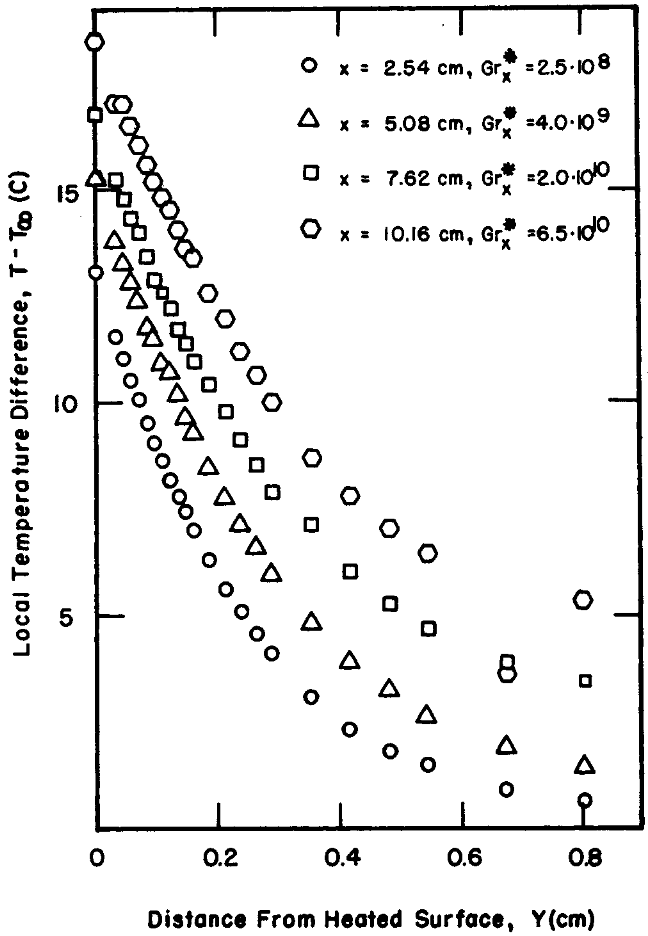

Fig. 5. Temperature profiles for vertical channel, W/L = 0.25, at four vertical positions.

* More detailed information regarding the experimental apparatus may be obtained by writing Department of Mechanical Engineering, Oregon State University, Corvallis, Oregon 97331.

files are presented in Figure 4 for the single plate and in Figure 5 for the narrowest channel spacing, in terms of $T - T_\infty$. It was observed that, for a given vertical location, the wall temperature decreased as the channel spacing decreased. Also, at a horizontal location of 0.8 cm, the temperatures for all vertical locations were more nearly equal for the single plate than for any of the channel configurations, which is demonstrated by comparing Figures 4 and 5. A continuation of this trend at greater distances from the vertical plate indicated that there was very little temperature stratification within the mercury tank itself as observed with the single plate, but that there was considerable stratification at the mid-point of each of the channel configurations.

For the purposes of comparison with analytical results, the velocity and temperature profiles are presented in dimensionless form for the single vertical plates in Figures 6 and 7. The similarity parameters of Sparrow and Gregg for position η, dimensionless temperature θ, and dimensionless velocity f' have been used. It appears that the similarity assumption holds quite well with the exception that the peak velocities increase with increasing vertical position. This is consistent with trends reported by Chang et al. (1964) from perturbation analysis.

Finally, the characteristics of the instability of the flow were measured. Figure 8 shows reproductions of the instantaneous anemometer output for the three different channel spacings and the four vertical locations used for the velocity profiles. Once again the heat flux was 3.31 × 10^4 J/m^2 s, thus the local Grashof numbers were the same as in Figure 3. All plots were obtained at a horizontal location of 0.38 cm. From Figure 3, it can be observed that this is approximately three times as far from the wall as is the velocity peak. This region was selected for comparison because the fluctuations were near their maximum amplitude, and changes in this amplitude occurred over a large distance; therefore the results are relatively insensitive to the actual horizontal location.

Note in Figure 8 that, at the 2.54-cm position, although there is some slight variation in the velocity, it is really very stable—the flow here is laminar. As the vertical position is increased to the 5.08-cm position the velocity becomes more disturbed, at 7.62 cm it is very unstable, and at 10.16 cm the fluctuations are even larger in amplitude and have a higher frequency. Comparing plots at 7.62 cm and 10.16 cm, the fluctuations are seen to be more regular in shape at the lower of the two positions. These distorted sinusoidal fluctuations are characteristic of unstable laminar flow. At the 10.16-cm position it appears that there are

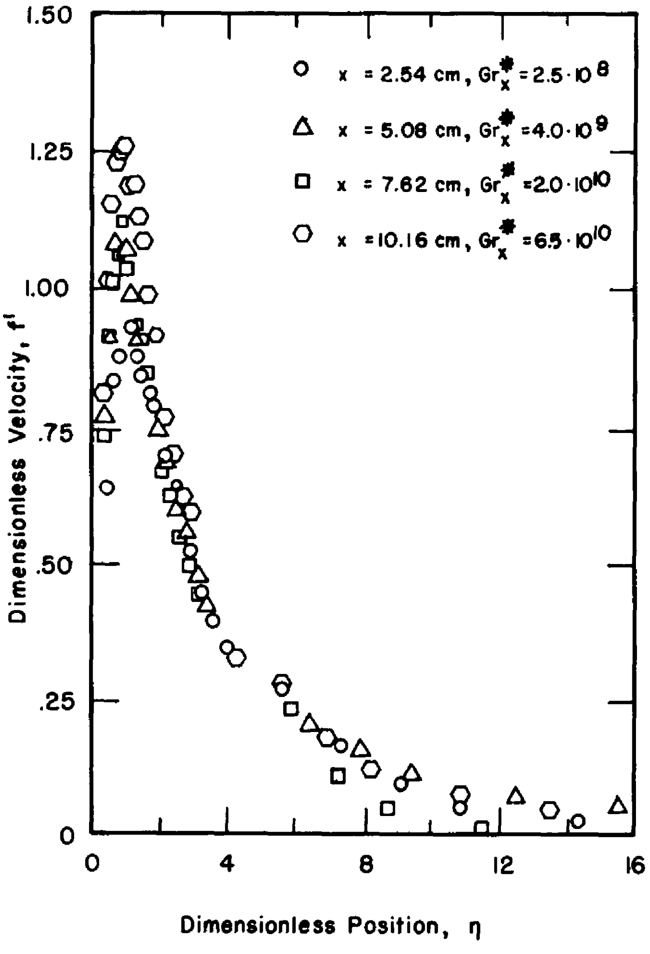

Fig. 6. Dimensionless velocity, single vertical plate.

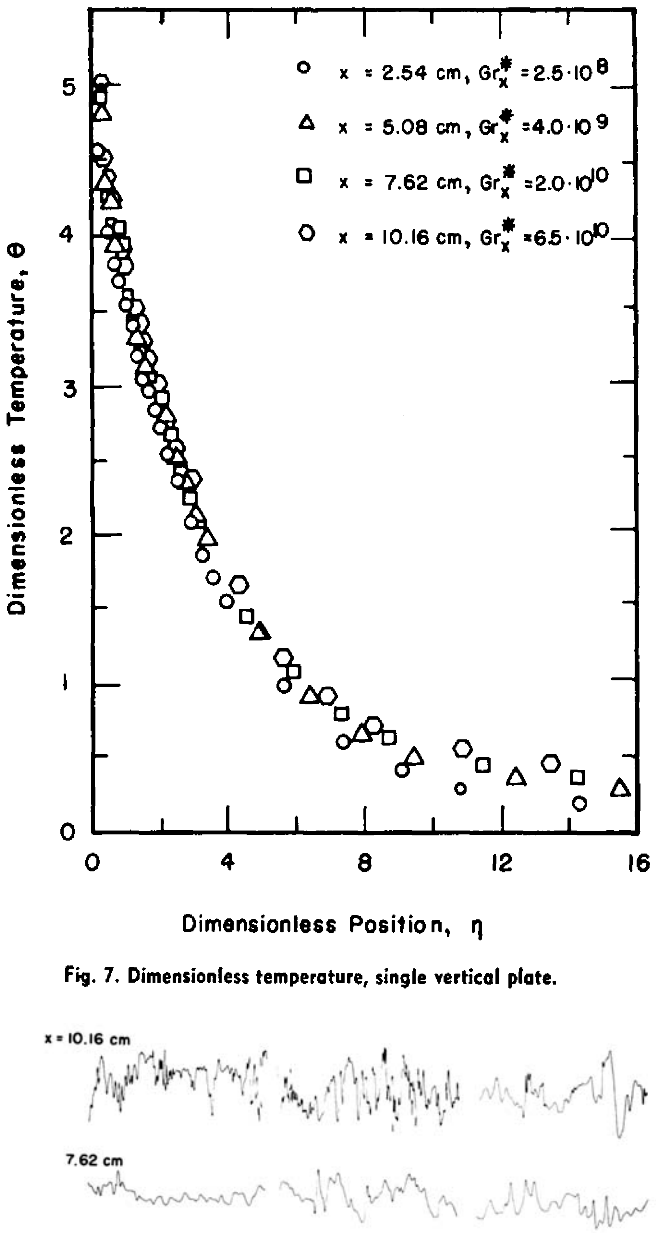

Fig. 7. Dimensionless temperature, single vertical plate.

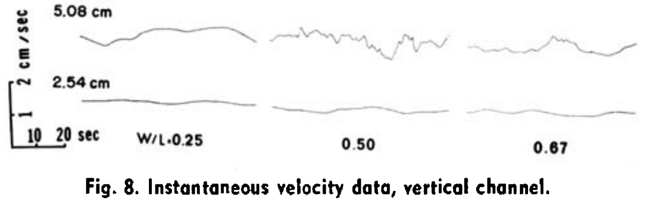

Fig. 8. Instantaneous velocity data, vertical channel.

intermittent periods of a higher frequency disturbance superimposed on these sinusoidal fluctuations. The intermittent higher frequency disturbances are turbulent bursts; therefore the flow regime is progressing from unstable laminar to turbulent, the common designation being transition. Note, finally, that the disturbances are generally greater and more advanced at the intermediate spacing, $W/L = 0.50$. It appears there is an optimum spacing in the area of 0.50 which is favorable to the onset of transition.

Heat Transfer Results

The heat transfer results can be divided into the two cases studied: (1) natural convection from a vertical plate with uniform heat flux, and (2) natural convection in a uniformly heated vertical channel. The results from the first case are applicable to analytical studies and are presented first.

Heat transfer data for the single vertical plate were taken by reading wall temperatures at intervals of 0.286 cm from the leading edge up to a vertical position of 10.43 cm for four different heat fluxes ranging from 2.49×10^4 J/m^2 s to 3.88×10^4 J/m^2 s. This produced a modified Grashof number range of approximately 10^6 to 10^{11}. The dimensionless heat transfer coefficient, the Nusselt number, was calculated using the measured heat flux and temperature difference, that is,

$$Nu_x = \frac{qx}{k(T_w - T_\infty)} \qquad (16)$$

The property values for the Nusselt and Grashof numbers were evaluated at the reference temperature recommended by Sparrow and Gregg (1958), $T_r = 0.7T_w + 0.3T_\infty$. All data were reduced using a CDC 3300 digital computer.

The heat transfer data from the single vertical plate are plotted in Figure 9. Note that the data are very linear on log-log coordinates thus can be well represented by a power expression. The data were regressed to obtain the least squares best fit using SIPS (Statistical Interactive Programming System), available on the CDC 3300 computer. SIPS uses a standard numerical scheme to solve the statistical normal equations and has check systems to prevent computer generated error. The resulting heat transfer expression, shown in Figure 9, is

$$Nu_x = 0.196\ Gr_x^{*0.188}, \quad 10^6 < Gr_x^* < 10^{11} \qquad (17)$$

Also shown in Figure 9 are the experimental results of Julian and Akins (1969), equation (13), and that of Colwell and Welty (1973), equation (14). Note that Equation (17) applies for a higher modified Grashof number

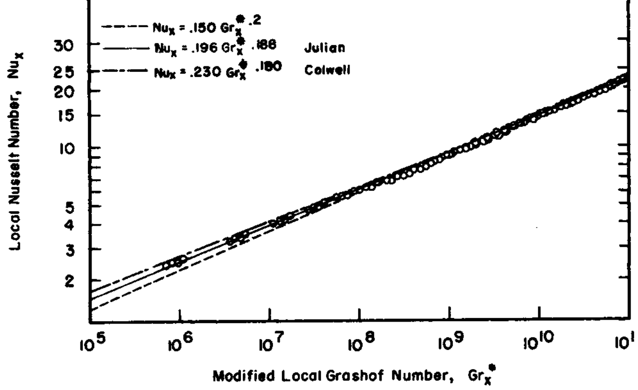

Fig. 9. A comparison of single plate correlations with other experimental work.

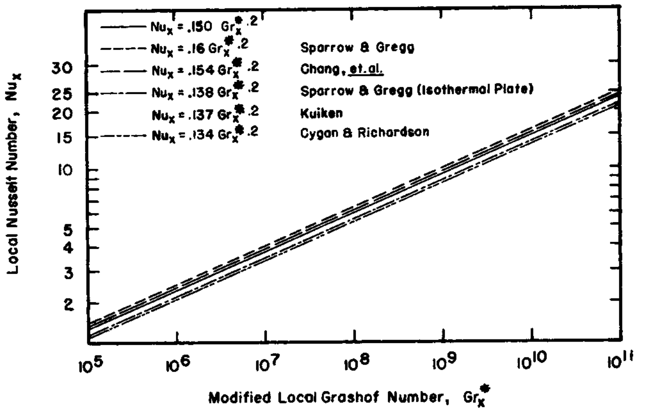

Fig. 10. A comparison of single plate correlations with analytical solutions.

range than Equations (13) and (14) which were for laminar flow. Note further that Equations (13) and (17) are otherwise identical.

In comparing the present results with analytical solutions, we note that all analytical solutions have an exponential power of 0.2 whereas the data regression produced a power of 0.188. Since there is little difference in the actual line for the applicable range between powers of 0.2 and 0.188, it is highly advantageous for comparison purposes to force the data regression line to have a power of 0.2. When this was done, the resulting correlation was

$$Nu_x = 0.150\ Gr_x^{*0.2} \qquad (18)$$

Equation (18) is shown in Figure 10 along with the analytical results for a uniform flux surface of Sparrow and Gregg (1956) equation (7), of Chang et al. (1964) equation (11), and with the analytical results for the isothermal surface of Sparrow and Gregg (1958) equation (8), Cygan and Richardson (1968) equation (11), and Kuiken (1969) equation (12). As can be observed from Figure 10, the present experimental results are slightly lower than the uniform flux solutions and somewhat higher than the isothermal surface solutions. One conclusion that can be drawn here is that, although the modeling assumptions for the analytical solutions included that of laminar flow, the resulting correlations still apply at a modified Grashof number of 10^{11}, where the flow is in transition.

The second configuration used in the heat transfer study was a uniformly heated vertical channel, where channel width-to-height ratios W/L of 0.25, 0.50, and 0.67 were used. Data were taken at three different heat flux levels between 2.94×10^4 J/m^2 s and 3.88×10^4 J/m^2 s for vertical positions ranging at intervals of 0.286 cm between 2.54 cm and 10.43 cm so that the modified Grashof number ranged from 10^8 to 10^{11}. The data were reduced and regressed in the same way as for the single vertical plate.

The reduced data for the uniformly heated vertical channel are plotted in Figure 11. When the results from the three different channel spacings are compared along with their respective regression equations, for most practical purposes, the results and the regression lines do not vary. All the data, therefore, were lumped into a single regression and the resulting heat transfer expression determined as

$$Nu_x = 0.252\ Gr_x^{*0.178}, \quad 0.25 < W/L < 0.67, \quad 10^8 < Gr_x^* < 10^{11} \qquad (19)$$

From a statistical point of view, using the F statistic, the regression lines for the three channel spacings are found to be significantly different. When the three lines

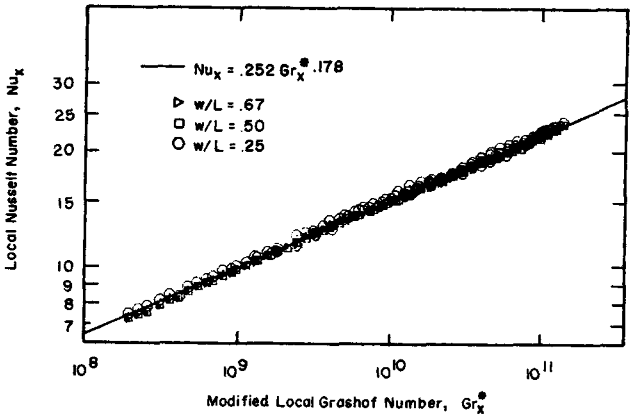

Fig. 11. Data and regression fit for all vertical channel cases.

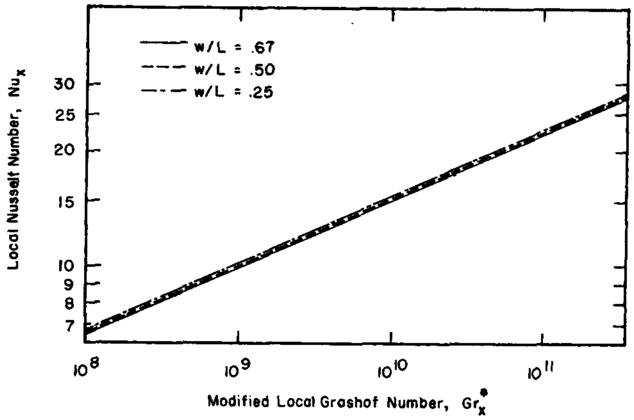

Fig. 12. Regression lines for vertical channel results at W/L = 0.67, 0.50, and 0.25.

are graphed, they appear to be nearly parallel with the Nusselt number increasing as the channel spacing W/L decreases from 0.67 to 0.25. This trend can be seen more clearly if, during the regression, the three regression lines are forced to be parallel by keeping the slopes equal but offset from each other by allowing the intercepts to vary. The resulting heat transfer expressions are

$$W/L = 0.25, \quad Nu_x = 0.2556\ Gr_x^{*0.178} \tag{20a}$$

$$W/L = 0.50, \quad Nu_x = 0.2518\ Gr_x^{*0.178} \tag{20b}$$

$$W/L = 0.67, \quad Nu_x = 0.2495\ Gr_x^{*0.178} \tag{20c}$$

in all cases:

$$10^8 < Gr_x^* < 10^{11}$$

Equations (20a), (20b), and (20c) are shown in Figure 12 for comparison. This trend is very interesting because it is consistent, at a higher Grashof range, with the results of Colwell and Welty (1973). Colwell and Welty found this effect to be most pronounced at channel spacings W/L near 0.10 with only a slight trend for spacings greater than 0.25. It is not surprising, then, that the trend observed here is very slight; nevertheless, it does indeed exist.

ACKNOWLEDGMENT

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NOTATION

C_p = specific heat at constant pressure

f' = dimensionless velocity, similarity,

$$f' \equiv \frac{ux}{5\nu(Gr_x^*/5)^{0.4}}$$

Gr_x^* = local modified Grashof number, $Gr_x^* \equiv \dfrac{\beta g \rho^2 x^4 q}{\mu^2 k}$

h = convective heat transfer coefficient, $h \equiv q/(T_w - T_\infty)$

k = thermal conductivity

L = vertical length of channel or plate

Nu_x = local Nusselt number, $Nu_x \equiv (hx)/k$

Pr = Prandtl number, $Pr \equiv (\mu C_p)/k$

q = heat flux

T = general temperature

T_r = reference temperature, $T_r = 0.7T_w + 0.3T_\infty$

T_w = wall temperature

T_∞ = ambient temperature

u = x-component of velocity

v = y-component of velocity

W = width of channel, distance between two heated surfaces

x = direction of mean flow, vertical

y = direction normal to plate surface, horizontal

Greek Letters

β = coefficient of thermal expansion,

$$\beta \equiv -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_p$$

η = dimensionless independent variable, similarity,

$$\eta \equiv \frac{y}{x}\left(\frac{Gr_x^*}{5}\right)^{0.2}$$

θ = dimensionless temperature, similarity,

$$\theta \equiv \frac{(T - T_\infty)k(Gr_x^*/5)^{0.2}}{xq}$$

μ = absolute viscosity

ρ = density

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